POWERS AND ROOTS

Powers:

We use powers to multiply a number by itself several times. A power has two parts, called base and exponent (index or power)

$\mathbf{a^n}$ where **a** is the base and **n** is the exponent,
and we read it "**a** to the power **n**" or "**a** to the **nth** power" or "a **raised** by the exponent of **n**" or "**a** to the **n**"

and it means that we multiply the base **a** by itself as many times as the index says

If the exponent is 2 we call it **squared**
If the exponent is 3 we all it **cubed**

-) Any number raised to the index one equals itself (the same number)

-) ANY NUMBER RAISED TO THE INDEX ZERO EQUALS ONE

-) The value of a power with base 10 is an 1 followed by so many 0 as the index says.

-) The squares of the integer numbers are called **perfect squares**. For instance, 81 is a perfect square, because $9^2 = 81$

Operations with powers:

-) Multiplication: To multiply two powers **with the same base**, we keep the base and we **add the indices.**

$$a^m \cdot b^n = a^{m+n}$$

-) Division: To divide two powers **with the same base**, we keep the base and we **subtract the indices.**

$$a^m : b^n = a^{m-n}$$

-) Addition and subtraction: We can't add or subtract powers, no matter what their bases are. We have to transform them into numbers first and then we do the operations.

-) Power: To raise a power to another power, we have to **multiply** the indices.

$$(a^n)^m = a^{n \cdot m}$$

-) Powers with the same index: When we have two powers with the same index but different bases, we can group them together by multiplying or dividing the bases and leaving the same index. This property works in both ways and it can be written as

$$(a \cdot b)^n = a^n \cdot b^n$$
$$(a : b)^n = a^n : b^n$$

So the power of a product is the product of the powers of the terms, and the power of a quotient is the quotient of the powers of the terms.

Scientific notation:

When we have a very big number that we can't use as it is, we will first round it to two or three significant figures and then write it as a product of a number and a power of the base ten. This is called scientific notation or standard form and it can also be used for very small numbers. The number will then have the form

$$a \cdot 10^n$$

where a is a number between 1 and 10, called mantissa or significand. n is another natural number that lets us know about the order of the number. The bigger the value of n, the bigger the number.

Notice that each time that we add one unit to the index, we are multiplying by 10 (remember the laws of powers)

Roots:

The opposite of squaring a number is called finding the **square root.**

The opposite of cubing a number is called finding the **cube root.**

Ex: The square root of 16 is 4 because $4^2 = 16$

The cube root of 8 is 2 because $2^3 = 8$

In general we will say that $\sqrt[n]{b} = a$ if $a^n = b$ and we will read "a is the nth root of b"

-) Perfect squares are the numbers that have an exact square root.

-) Whole square roots or **integral roots** are approximations of the values of a root when they are not exact. For instance, the whole square root of 50 is 7, because $7^2 = 49$ is the closest perfect square.

-) Any positive number has two square roots with the same absolute value but different signs. A negative number has no square roots.

1) Write the name of the following powers:

a) 5^2 ____

b) 7^3 ____

c) 2^5 ____

d) 9^{18} ____

2) Find the value of these powers. Express the answer with words.

a) Five cubed is ____

b) Three to the fourth power is _____________________________

c) Seven squared is ______________________________________

d) Ten to the power five is ______________________________

e) Thirty-two to the power one is ___________________________

f) Eighty to the power zero is ____________________________

g) Two to the power six is _______________________________

3) Find the value of these roots. Express the answer with words.

a) The square root of 36 is ________________________________

b) The cube root of 125 is ______________________________

c) The square root of 10000 is _____________________________

d) The square root of 400 is _______________________________

e) The cube root of 8 is ___________________________________

4) Find a perfect square between 450 and 500

5) Write the squares of all numbers between 1 and 15

6) Write these numbers using scientific notation:

a) 42 million b) 632000 c) 50783100

7) Without a calculator, write the whole number that's closest in value to

a) $\sqrt{50}$ b) $\sqrt{157}$ c) $\sqrt{67}$

Remember that this is called the integral root

8) Work out the value of the following expressions:

a) $3^2+\sqrt{20+5}=$

b) $3\cdot 2+\sqrt{16}:\sqrt{4}=$

c) $2+30:15-(\sqrt{25}+15)+2\sqrt{9}=$

d) $(4+1)\cdot\sqrt{36}-(2-1)^3=$

e) $2\cdot 3^2-3\cdot\sqrt{64}+7\cdot 0+1-4=$

f) $2+5\cdot 2^3-(\sqrt{25}-\sqrt{9})^2=$

9) Write the value of the following numbers:

a) $4 \cdot 10^5 + 2 \cdot 10^4 + 7 \cdot 10^3 + 5 \cdot 10^2 + 6 \cdot 10 + 1 =$

b) $10^7 + 9 \cdot 10^5 + 5 \cdot 10^2 + 3 =$

10) A cell divides in half every hour to form two new cells. How many cells do we have ten hours later?

11) In a warehouse we have placed 12 rows of squared boxes making a square. How many boxes do we have?

12) Find the side of a square with an area of 81 cm^2.

13) I have 196 small cubes with the same size and I want to form a square with them. How many cubes do I have to place in each side?

14) If I have 425 cubes instead, how many cubes do I need now to form each side? How many cubes are left? Could I construct another square with them?

15) A square field has a surface of 900 m^2. How many meters of wire mesh do we need in order to round it? If every meter costs €1.5, how much money will it cost?

16) An old legend tells us about the invention of chess. It is said that the sultan was so happy with the new game that he told the inventor that he could ask for anything he wanted in return. So the inventor answered that he wanted a grain of wheat on the first square, two grains on the second, four grains on the third.... doubling the number of grains on each square. How many grains of wheat will we have on the last square? Round the number to two significant figures and use scientific notation.
Note: a chessboard has 64 squares.