UNIT 2: POWERS AND ROOTS

Exercise 1: Work out the value of the following powers:

a) $-2^4 =$
b) $(-3)^3 =$
c) $7^{-2} =$
d) $5^{-1} =$
e) $\left(\frac{5}{7}\right)^{-2} =$
f) $\left(\frac{3}{4}\right)^{-3} =$
g) $(-5)^{-4} =$
h) $(-2)^{-5} =$

Exercise 2: Express as a single power:

a) $7^6 : (7^4 \cdot 7) =$
b) $(5^{12} : 5^7) : (5^2 \cdot 5^3) =$
c) $x^5 \cdot x^{-3} =$
d) $y^{-4} \cdot y^3 =$
e) $a^2 \cdot a^{-5} \cdot a =$
f) $(y^7 \cdot y^{-2}) : (y^{-3} \cdot y^{-5}) =$
g) $(5^3)^{-5} : (5 \cdot 5^4)^3 =$
h) $(x^5 : x^{-2}) : x^7 =$

Exercise 3: Express as a single power:

a) $(3^4)^{-2} \cdot 3^{10} =$
b) $x^2 \cdot x^{-5} : x^4 =$
c) $(7^{11} : 7^5) : (7^{10} \cdot 7^{-4}) =$
d) $(a^{-3} \cdot a^8) : (a^{-2} \cdot a^{-6}) =$
e) $(5^2 \cdot 5^{-7}) : (5^{-3} \cdot 5^{-4}) =$
f) $(x^8 : x^{-5}) : (x^{15} \cdot x^{-2}) =$
g) $\left(x^{-4} \cdot x^{-1}\right)^{-3} =$
h) $x^{-1} : (x^2 \cdot x^{-3}) =$

Exercise 4: Express as a product of powers:

a) $\frac{a^3 \cdot a \cdot b^{-5}}{a^{-2} \cdot b^6} =$
b) $\frac{a^{-7} \cdot b^4 \cdot a^8 \cdot b^{-6}}{a^{-2} \cdot b^3} =$
c) $\frac{x^{-5} \cdot y^4 \cdot x^3 \cdot y^{-7}}{y^2 \cdot x^{-9} \cdot y^{-8}} =$
d) $\frac{x^7 \cdot y^{-2} \cdot x^{-8} \cdot y^5}{y^7 \cdot x^3 \cdot y^{-4}} =$
e) $\frac{x^{-4} \cdot y^3 \cdot (y^2)^{-5} \cdot x^{14}}{x^2 \cdot y^{-4} \cdot x^3} =$
f) $\frac{a^3 \cdot b^{-2} \cdot b^{-7} \cdot a^{-5}}{a^{-5} \cdot b^{-7} \cdot a \cdot b^{-10}} =$

Exercise 5: Express as a single power:

a) $2^{-4} \cdot 2^{-3} : 2^{10} =$
b) $(y^7 \cdot y^{-9}) : (y^6 : y^8) =$
c) $(x^{-5})^{-2} \cdot (x^3 \cdot x^5) =$
d) $(x^8 \cdot x^{-5}) : x^2 =$
e) $(5^3)^{-5} : (5 \cdot 5^4)^3 =$
f) $(y^{-9} \cdot y^4) : (y^7 : y^2) =$

Exercise 6: Work out:

a) $(42^8 : 7^8) : (3^4 \cdot 2^4) =$
b) $(4^{-7} \cdot 5^{-7}) : (40^3 : 2^3) =$
c) $(18^5 : 3^5) : (3^7 \cdot 2^7) =$
d) $(2^{-5} \cdot 5^{-5}) \cdot (20^3 : 2^3) =$

Exercise 7: Express as a product of powers:

a) $\frac{5^4 \cdot 2^7 \cdot 10}{25^3 \cdot 2^2} =$

b) $\frac{28^5 \cdot 49^2}{2^4 \cdot 7^5} =$

c) $\frac{9^4 \cdot 3^2 \cdot 27}{3 \cdot 9^5} =$

d) $\frac{10 \cdot 5^3 \cdot 2^4}{20} =$

Exercise 8: Express as a product of powers:

a) $\frac{2^{-3} \cdot 8^2 \cdot 5^6}{25^{-2} \cdot 16} =$

b) $\frac{18^{-3} \cdot 3^2 \cdot 2}{12^{-5}} =$

c) $\frac{27^{-4} \cdot 3^5 \cdot 12^2}{18^{-5}} =$

d) $\frac{15^3 \cdot 3^{-7} \cdot 5^4}{5^2 \cdot (3^2)^{-3}} =$

Exercise 9: Work out:

a) $(x^3 \cdot x^{-7}) : (x^8 \cdot x^{-12}) =$

b) $(y^7 : y^{-5}) \cdot (y^2 : y^{15}) =$

c) $(x^5 \cdot x^{-2} \cdot x) \cdot (x^4 : x^8) =$

d) $(y^7 \cdot y^{-2}) : (y^{-3} \cdot y^{-5}) =$

e) $(a^2)^{-5} : (a^7 \cdot a^2)^2 =$

f) $(a^{-5} \cdot a)^3 \cdot (a^2 : a^{10}) =$

Exercise 10: Work out:

a) $\frac{a^3 \cdot a \cdot b^{-5}}{a^{-2} \cdot b^5} =$

b) $\frac{2^{-7} \cdot 5^{-2} \cdot 2^{-4} \cdot 5^{-4}}{5^{-3} \cdot 2^9} =$

c) $\frac{x^4 \cdot y^3 \cdot x^{-7} \cdot y}{x^{-2} \cdot y^8} =$

d) $\frac{x^{-2} \cdot y^3 \cdot x^{-9}}{x^{-5} \cdot y^8 \cdot x^{-6}} =$

Exercise 11: Work out:

a) $\frac{16^2 \cdot 2^{-3} \cdot 5^4}{20^{-4} \cdot 2 \cdot 5^{-3}} =$

b) $\frac{27^{-4} \cdot 3^5 \cdot 5^{-3}}{15^2 \cdot 3} =$

c) $\frac{7^4 \cdot 4^{-3} \cdot 28}{(2^5)^{-3} \cdot (7^4)^2} =$

d) $\frac{15^{-3} \cdot 27^8}{3^4 \cdot 5^{-9}} =$

Exercise 12: The prefix tera in terabyte represents one (Spanish) billion of bytes. Write this prefix as a power of ten.

Exercise 13: A modern work of art represents a tree where every branch divides in three ad infinitum, fractal-like. If five branches sprout from the trunk, how many branches are there in the seventh level of the tree?

Exercise 14: Let's create a chain where a piece of fake news is spread around. I invent a gossip about one of my coworkers and sent it via whatsapp to five of my contacts. Each one of them will resend the story to five new contacts of theirs, and so on. How many steps do we need until all Spain knows it, even if they don't know my coworker?

Exercise 15: An old legend tells us about the invention of chess. It is said that the sultan was so happy with the new game that he told the inventor he could ask for anything he wanted in return. So the inventor told him he wanted a grain of wheat on the first square, two grains on the second, four grains on the third.... doubling the number of grains on each square. How many grains of wheat will we have on the last square? Round the number to three significant figures and use scientific notation.
Note: A chessboard has 64 squares.

Exercise 16: Work out the value of the following roots:

a) $\sqrt{1764} =$

b) $\sqrt{2401} =$

c) $\sqrt{422500} =$

d) $\sqrt[7]{2^7 \cdot 5^7 \cdot 7^7} =$

e) $\sqrt[3]{2^6 \cdot 3^{12} \cdot 5^{15}} =$

f) $\sqrt{2^{12} \cdot 3^4 \cdot 5^2 \cdot 7^{10}} =$

g) $\sqrt[3]{125000} =$

h) $\sqrt[5]{759375} =$

i) $\sqrt[7]{\dfrac{a^{14} b^{42}}{c^{63}}} =$

j) $\sqrt[3]{x^9 y^6 z^{-12}}$

Exercise 17: Work out the value of the following roots:

a) $\sqrt{49\,000\,000\,000\,000} =$

b) $\sqrt[5]{x^{15} y^{-5} z^{-35}} =$

c) $\sqrt[4]{625\,000\,000\,000\,000} =$

d) $\sqrt[7]{\dfrac{a^7 b^{-14}}{c^{-21}}} =$

e) $\sqrt[4]{5062500000000} =$

f) $\sqrt{\dfrac{u^{-10} v^{12}}{w^{18} z^{-6}}} =$

Exercise 18: Work out the value of the following expressions:

a) $2+3\sqrt{49}-(\sqrt{36}:2)^2-2\cdot(8-5)^2 =$

b) $3-(-5)\cdot(-2)+\sqrt{12+4}:(-2)+6\cdot 2^2 =$

c) $3-(-15):(+3)-(-5)^2+\sqrt[3]{36-4} =$

d) $(-2)^3-\sqrt{49}:(-7)-3^2-(-5)\cdot(-2) =$

e) $5-7\cdot(-4)+\sqrt{36}:(-3)-(3-5)^3 =$

Exercise 19: Work out the value of the following expressions:

a) $(-1)^5+(10-2\cdot 7)^2-\sqrt{49}\cdot(-2) =$

b) $6-4\cdot 3-\sqrt{16}\cdot(-7)-5\cdot 2^3+(-3)^2 =$

c) $(\sqrt{81}-\sqrt{64})^5-\sqrt{36}:(-6)-5^2+(-20):(-2) =$

d) $\sqrt{4+5\cdot 3^2}:7+20:\sqrt{10+6}-(-1)^3 =$

e) $-5+2\cdot\sqrt{25}-(-1)^{27}+(\sqrt{81}-\sqrt{36})^2+2^3 =$

Exercise 20: Find the length of the side of a square with an area of 20736 cm^2.

Exercise 21: I have 2744 small cubes of the same size and I want to form a big cube with them. How many cubes do I have to place in each side?

Exercise 22: If I had 3718 cubes instead, how many cubes do I need now to form each side? How many cubes are left? Could I construct another cube with them?
Hint: Find the value of the different powers, since you cannot use a calculator.

Exercise 23: I want to build a square pen for my sheep so they don't go away. The area of the field is 3136 m^2. How many meters of fence do I have to use? If every meter costs €3.75, how much money will it cost?

Exercise 24: I have 75625 tiles and I want to pave the biggest possible square with them. How many tiles are there on every side? How many tiles are left?

Exercise 25: Work out and express as a product of powers:

a) $\frac{100^3 \cdot 75}{16 \cdot 2 \cdot 5^2} =$

b) $\frac{25^4 \cdot 20 \cdot 2^5}{4 \cdot 5^3} =$

c) $\frac{2^3 \cdot 8^3 \cdot 4^6}{16 \cdot 32} =$

d) $\frac{12^5 \cdot 3^2 \cdot 2^4}{6^3 \cdot 3} =$

Exercise 26: Work out:

a) $5+4\cdot\sqrt{25}-(\sqrt{36}-\sqrt{9})^2+(5-3)^3 =$

b) $8:(\sqrt{17-1}-\sqrt{4})+3\cdot 5^2-\sqrt{98:2} =$

c) $3+4\cdot\sqrt{25}-1^{72}+\sqrt{85-4} =$

d) $7+2\cdot\left(\sqrt{64}-\sqrt{25}\right)^2-12:\sqrt{9}+7 =$

e) $5+3\sqrt{30-5}+(\sqrt{64}-\sqrt{36})^3-\sqrt{100}:\sqrt{25} =$

Exercise 27: Work out:

a) $a^{-2}\cdot a^7 : a^5 =$

b) $\left(x^4\right)^{-3} : x^7 =$

c) $a^2 \cdot a^{-5} : a^{-7} =$

d) $e^{-12} : e^{-7} \cdot e^{-1} =$

e) $v^{-7} \cdot v^{-2} : v^5 =$

f) $\left(a^5 : a^{-5}\right)\cdot\left(a^7 : a^{-3}\right) =$

g) $\left(w^{-7} \cdot w^{-5}\right) : \left(w^3 \cdot w^{-8}\right) =$