EQUATIONS - FUNCTIONS

3º ESO

Exercise 1: (2.5 points)

a) Find the value of k so that when dividing the polynomial $P(x)=x^3+kx^2+3x+7$ by $(x+2)$ the remainder is 13 $\rightarrow$ $k=5$ (0.75)

b) Divide (x^4+5x^2-3x+4) by (x^2-3) $\rightarrow$ $\begin{cases}\text{Quotient: } x^2+8\\ \text{Remainder: } -3x+28\end{cases}$ (1)

c) Divide (x^4+7x^3-4x+1) by $(x-2)$ $\rightarrow$ $\begin{cases}\text{Quotient: } x^3+9x^2+18x+32\\ \text{Remainder: } 65\end{cases}$ (0.75)

Exercise 2: (3 ptos) Factorize the following polynomials and indicate their roots:

a) $P(x)=x^4+x^3-12x^2+4x+16$ $\rightarrow$ $\begin{cases}\text{Roots: } x=-1,\ x=2 \text{ double},\ x=-4\\ \text{Factorization: } (x+1)(x-2)^2(x+4)\end{cases}$ (1.25)

b) $Q(x)=x^3+2x^2+4x+8$ $\rightarrow$ $\begin{cases}\text{Roots: } x=-2\\ \text{Factorization: } (x-2)(x^2+4)\end{cases}$ (0.75)

c) $R(x)=x^6-29x^4+100x^2$ $\rightarrow$ $\begin{cases}\text{Roots: } x=0 \text{ double},\ x=\pm 2,\ x=\pm 5\\ \text{Factorization: } x^2(x+2)(x-2)(x+5)(x-5)\end{cases}$ (1)

Exercise 3: (1 pto) I've factorized the polynomial $P(x)$ and I got

$$P(x)=3x^4+x^3+2x^2+5=x(x+2)(x+1)^2(x-5)$$

Find at least five mistakes

-) The degree of the original polynomial is 4, while the factorization has a degree of 5

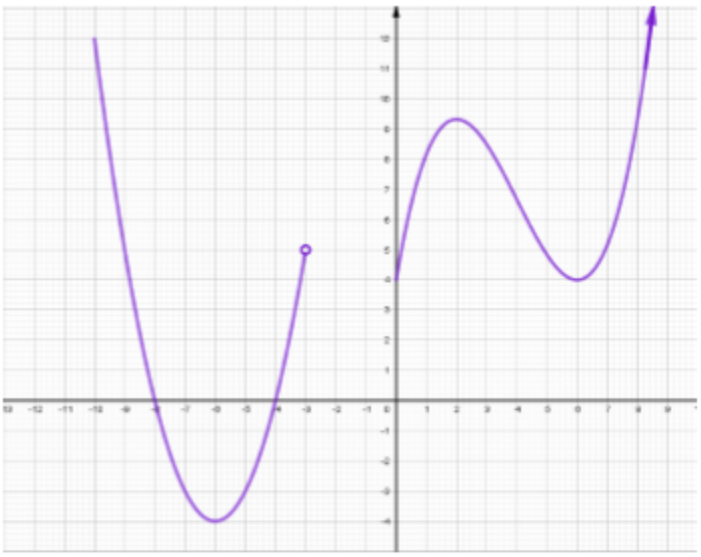
-) There must be a 3 at the beginning of the factorization, and some of the roots are fractions.

-) 2 is not a divisor of 5, so it cannot be a root, either positive or negative

-) All the terms are positive, so the roots must be negative $\rightarrow$ $(x-5)$ is not a factor

-) You cannot take x as a common factor

Exercise 4: (2 points) Given the following graph of a certain function:



a) Indicate its domain and its image

$\text{Dom } f = [-10,-3) \cup [0,+\infty)$

$\text{Im } f = [-4,+\infty)$

b) Determine the points where the function crosses the axes

$\underline{OX}$ $x=-8,\ x=-4$

$\underline{OY}$ $y=4$

c) Study its monotony

Increases: $(-6,-3)$ and $(0,2)$ and $(6,+\infty)$

Decreases: $(-10,-6)$ and $(2,6)$

d) Study the extrema

Relative maxima: $x=-10,\ x=2$ $\rightarrow$ Absolute maximum: $\nexists$

Relative minima: $x=-6,\ x=0,\ x=6$ $\rightarrow$ Absolute minimum: $x=-6$

Exercise 5: (1.5 points) Indicate the domain of the following functions:

a) $f(x)=\dfrac{x^2-1}{x^2-9x}$ $\rightarrow$ $\text{Dom } f = \mathbb{R}-\{0,9\}$

b) $f(x)=\sqrt[8]{x+7}$ $\rightarrow$ $\text{Dom } f = [-7,+\infty)$

c) $f(x)=\dfrac{5x+3}{\sqrt{x-5}}$ $\rightarrow$ $\text{Dom } f = (5,+\infty)$